

Handling of T-bills in Forecasting

Why T-bills are different

T-bills (for our purposes, 30-day Treasury bills) differ from other assets in a number of ways. For one thing, many of our models are based on excess return over the T-bill rate. For another, T-bill returns are relatively more predictable. In fact, when a T-bill matures on the final day of the first forecast month, we know exactly what the return over that month will be. Even when no T-bill matures exactly on that date, the known T-bill yield at the start of the month, adjusted for maturity date, is a very good predictor of T-bill return for the month. Hence it makes sense to treat T-bills differently.

What the user forecasts versus what the software forecasts

The software computes forecasts for asset mean returns, variances, and covariances based on a user's input forecast that may cover only the first part of the future reporting period and only some of the factors. There may not be a forecast at all.

When the user forecasts interest rates for a given month, the user is forecasting the yields as of the *end* of the forecast month in question. (Note that in the first forecast month we already know what the yields are at the *start* of the month, so it makes sense for the forecast to refer to the month-end yield.) The forecast change in yields over a month is what permits us to compute implied realizations for the first two macroeconomic factors. (The details of this need not concern us here.)

As noted in the previous section, the yield for 30-day T-bills at the end of a month is effectively the return for the *following* month. Hence when a user forecasts T-bill yield for the end of, say, August, he or she is implicitly forecasting the return in the following month, September. Furthermore, whether we have a forecast or not, the return for the first forecast month is known (at least to a very good approximation) because we know the yield at the end of the previous month.

Based on the user's forecast and on historical data, the software computes a mean factor return and factor covariance matrix for each future month over the number of months requested by the user. This information, combined with asset risk exposures (Betas) is used to compute asset returns and covariances for all the assets in the portfolio.

It is probably worth stating for completeness that in computing VaR and Tracking Error, the software makes use not just of individual months but of n -month averages. The VaR for the next month is based on the forecasted asset means and covariances for the first forecast month. The VaR for a two-month future holding period is based on the asset means and covariances for the first and second forecast month. (The two monthly means vectors and covariance matrices are actually averaged together, element-by-element, to create a means vector and covariance matrix for the two-month forecast period.) The VaR for a three-month holding period is based on averaging means and covariances for the first, second, and third months, and so on.

The forecasted asset mean returns and asset covariances output by BIRRSMV represent the average returns and averages covariances over the number of forecast months requested by the user.

T-bill return forecast

As previously noted, the T-bill return for the first forecast month is already in the database. It's simply the one-month T-bill yield known at the historical sample period.

If the user forecasts interest rates, then each T-bill interest rate (yield) forecasted by the user constitutes a forecast of T-bill return for the *following* month, for the reasons explained above. Since the user enters the yield in compounded annualized units, that value is converted to a monthly equivalent by the formula $(1+TBY)^{-12} - 1$.

The requested output forecast period might well include months for which there is no known or user-forecasted T-bill return. We estimate a T-bill forecast for those months as follows: First, we perform an ordinary least squares regression for T-bill returns over a 72-month sample period (ending in the first forecast month) using the autoregressive model $TB(t) = a + b \cdot TB(t-1) + \varepsilon(t)$ to obtain estimates of the parameters a and b . These OLS estimates are denoted by \mathbf{a} and \mathbf{b} , respectively. We then generate any needed T-bill forecast months by applying the formula $TB(t) = \mathbf{a} + \mathbf{b} \cdot TB(t-1)$ iteratively starting in the first month without a T-bill forecast.

Computational treatment of T-bills in forecasting asset returns

Just as a T-bill yield forecast for month n implies a return forecast for month $n + 1$, likewise a T-bill return forecast for month $n + 1$ implies a yield forecast for month n . Since the regression mentioned in the previous section gives us a T-bill return forecast for every future month not already forecasted, we could say that we implicitly have a T-bill yield forecast in every future month as well. However, we do not have a yield forecast for long-term corporate or government bonds, so we make no use of that implicit yield forecast in connection with the interest rate factors.

Instead, the T-bill forecast is used only in calculating the total forecast return for an asset, given a forecast of its excess return over T-bills (the computation of which is described separately).