

*Using Macroeconomic Factors to Control Portfolio Risk*¹

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I. Introduction

A fundamental principle of finance is the tradeoff between risk and return. Unless a portfolio manager possesses special information, one portfolio can be expected to outperform another only if it is riskier in some appropriate sense. The crucial question is: “What is the appropriate measure of risk?”

There are many attributes that might be related to an asset’s risk, such as market capitalization (size), dividend yield, growth, price/earnings ratio, and so on. But there are at least three problems with these traditional descriptors:

- (i) Most are based on accounting data, and such data are generated by rules which may differ significantly across firms.
- (ii) Even if all firms used the same accounting rules, reporting dates differ so that it is difficult to construct time-synchronized inter-firm comparisons.
- (iii) Most importantly, there is no rigorous theory to tell us how traditional accounting variables **should** be related to an appropriate measure of risk for computing the risk-return tradeoff. Even if historical empirical relationships can be uncovered, without the

foundation of a rigorous theory one must be concerned that any historical correlation might be spurious and subject to sudden and material change.

Currently there are only two theories that provide a rigorous foundation for computing the tradeoff between risk and return:

- (i) the Capital Asset Pricing Model (CAPM), and
- (ii) the Arbitrage Pricing Theory (APT).

The CAPM, for which William F. Sharpe shared the 1990 Nobel Memorial Prize in Economics, predicts that only one type of nondiversifiable risk influences expected security returns, and that single type of risk is “market risk.”² In 1976, a little more than a decade after the CAPM was proposed, Stephen A. Ross invented the APT. The APT is more general than the CAPM in accepting a variety of different risk sources. This accords with the intuition that, for example, interest rates, inflation, and business activity have important impacts on stock return volatility.

While some theoretical formulations of the APT can be more intellectually demanding than the CAPM, the intuitively appealing basics behind the APT are easy to understand. Moreover, the APT provides a portfolio manager with a variety of new and easily implemented tools to control risks and to enhance portfolio performance.

The remainder of this paper is organized as follows. First, we will explain APT basics and the equations of the APT. Second, we will discuss macroeconomic forces that are the underlying sources of risk. Third, we will illustrate some risk exposure profiles and the resulting APT-based risk-return tradeoffs, and we will show how these fundamental risks contribute to both the expected and unexpected components of realized return. Finally, we will discuss several uses of the APT that every practitioner could easily employ.

II. The APT Basics

Both the CAPM and the APT agree that, though many different firm-specific forces can influence the return on any individual stock, these idiosyncratic effects tend to cancel out in large and well-diversified portfolios. This cancellation is called the **principle of diversification**, and it has a long history in the field of insurance. An insurance company has no way of knowing whether any particular individual will become sick or will be involved in an accident, but the company is able to accurately predict its losses on a large pool of such risks.

However, an insurance company is not entirely free of risk simply because it insures a large number of individuals. For example, natural disasters or changes in health care can have major influences on insurance losses by simultaneously affecting many claimants. Similarly, large, well-diversified portfolios are not risk free because there are common economic forces that pervasively influence all stock returns and that are not eliminated by diversification. In the APT, these common forces are called **systematic** or **pervasive risks**.

According to the CAPM, systematic risk depends only upon exposure to the overall market, usually proxied by a broad stock market index such as the S&P 500. This exposure is measured by the **CAPM beta**, as defined in footnote 2. Other things equal, a “beta” greater (or less) than 1.0 indicates greater (or less) risk **relative** to swings in the market index.³

The APT takes the view that there need not be any single way to measure systematic risk. While the APT is completely general and does not specify exactly what the systematic risks are, or even how many such risks exist, academic and commercial research suggests that there are several primary sources of risk which consistently impact stock returns. These risks arise from **unanticipated** changes in the following fundamental economic variables:

- Investor confidence
- Interest rates
- Inflation
- Real business activity
- A market index

Every stock and portfolio has exposures (or betas) with respect to each of these systematic risks. The pattern of economic betas for a stock or portfolio is called its **risk exposure profile**. Risk exposures are rewarded in the market with additional expected return, and thus the risk exposure profile determines the volatility **and** performance of a well-diversified portfolio. The profile also indicates how a stock or portfolio will perform under different economic conditions. For example, if real business activity is greater than anticipated, stocks with a high exposure to business activity, such as retail stores, will do relatively better than those with low exposures to business activity, such as utility companies.

Most importantly, an investment manager can control the risk exposure profile of the managed portfolio. Managers with different traditional styles, such as a small-capitalization growth manager or a large-capitalization value manager, have differing inherent risk exposure profiles. For this reason a traditional manager’s risk exposure profile is congruent to a particular **APT style**.

Given any particular APT style (or risk exposure profile), the difference between a manager’s expected return and his or her actual performance is attributable to the selection of individual stocks which perform better or worse than their *a priori* expectations. This extraordinary performance defines *ex post* **APT selection**.

III. The Equations of the APT

This section explains the equations that constitute an APT model of asset returns. The APT follows from two basic postulates:

> Postulate 1:

The first postulate is that, in every time period, the difference between the actual (realized) return and the expected return for any asset is equal to the sum, over all risk factors, of the risk exposure (the beta for that risk factor) multiplied by the realization (the actual end-of-period value) for that risk factor, plus an asset-specific (idiosyncratic) error term. This postulate is expressed by equation (1):

$$(1) \quad r_i(t) - E[r_i(t)] = \beta_{i1} f_1(t) + \dots + \beta_{iK} f_K(t) + \varepsilon_i(t),$$

$$\left\{ \begin{array}{l} \text{systematic} \\ \text{risk} \end{array} \right\} \quad \left\{ \begin{array}{l} \text{idiosyn-} \\ \text{cratic risk} \end{array} \right\}$$

where

$r_i(t)$ = the total return on asset i (capital gains plus dividends) realized at the end of period t ,

$E[r_i(t)]$ = the expected return, at the beginning of period t ,

β_{ij} = the risk exposure or beta of asset i to risk factor j for $j = 1, \dots, K$,

$f_j(t)$ = the value of the end-of-period realization for the j -th risk factor,

$j = 1, \dots, K$, and

$\varepsilon_i(t)$ = the value of the end-of-period asset-specific (idiosyncratic) shock.

It is assumed that the expectations, at the beginning of the period, for all of the factor realizations and for the asset-specific shock are zero, i.e.,

$$E[f_1(t)] = \dots = E[f_K(t)] = E[\varepsilon_i(t)] = 0.$$

It is also assumed that the asset-specific shock is uncorrelated with the factor realizations, i.e.,

$$\text{cov}[\varepsilon_i(t), f_j(t)] = 0 \text{ for all } j = 1, \dots, K.$$

Finally, it is assumed that all of the factor realizations and the asset-specific shocks are uncorrelated across time, i.e.,

$$\text{cov}[f_j(t), f_j(t')] = \text{cov}[\varepsilon_i(t), \varepsilon_i(t')] = 0 \text{ for all } j = 1, \dots, K \text{ and for all } t \neq t'.$$

The above conditions are summarized by saying that asset returns are generated by a **linear factor model**. Note that the risk factors themselves may be correlated (inflation and interest rates, for example), as may the asset-specific shocks for different stocks (as would be the case, for example, if some unusual event influenced all of the firms in a particular industry).

> Postulate 2:

The second basic postulate of the APT is that **pure arbitrage profits are impossible**. That is, it is assumed that because of competition in financial markets, it is impossible for an investor to earn a positive expected rate of return on any combination of assets without undertaking some risk and without making some net investment of funds.

It should be stressed that Postulate 2 is in fact an appealing equilibrium concept that has far-ranging implications for broad areas of financial economics well beyond the determination of asset prices. It is hard to imagine any model of financial behavior which fails to conclude that pure arbitrage profits tend to zero. This generality brings many advantages. The APT is free of restrictive assumptions on preferences or probability distributions, and it provides a rigorous logical foundation for the tradeoff between expected returns and risks.

Given Postulates 1 and 2, the main APT theorem is that there exist $K+1$ numbers P_0, P_1, \dots, P_K , not all zero, such that the expected return on the i -th asset is approximately equal to P_0 plus the sum over j of β_{ij} times P_j , i.e.,

$$(2) \quad E[r_i(t)] \approx P_0 + \beta_{i1}P_1 + \dots + \beta_{iK}P_K.$$

Although equation (2) holds only approximately, with additional assumptions it can be proved that it holds exactly. More importantly, even without any additional assumptions, it has been proved that the approximation in equation (2) is sufficiently accurate that any error can be ignored in practical applications. Thus the approximation symbol, \approx , can be replaced by an equal sign, and we rewrite equation (2) as

$$(3) \quad E[r_i(t)] = P_0 + \beta_{i1}P_1 + \dots + \beta_{iK}P_K.$$

Here P_j is the **price of risk** or **the risk premium for the j -th risk factor**. These P_j 's determine, via equation (3), the risk-return tradeoff that we have been seeking.⁴

Imagine a portfolio which is perfectly diversified [i.e., one for which $\varepsilon_p(t) = 0$] and with no factor exposures [$\beta_{pj} = 0$ for all $j = 1, \dots, K$]; such a portfolio has zero risk, and from equation (3) its expected return is P_0 . Thus, P_0 must be the risk-free rate of return. Reasoning similarly, the risk premium for the j -th risk factor, P_j , is the return, in excess of the risk-free rate, earned on an asset which has one unit of risk exposure to the j -th risk factor ($\beta_{ij} = 1$) and zero risk exposures to all of the other factors ($\beta_{ih} = 0$ for all $h \neq j$).

The full APT is obtained by substituting equation (3) into equation (1), which after rearranging terms yields:

$$(4) \quad r_i(t) - P_0 = \beta_{i1}[P_1 + f_1(t)] + \dots + \beta_{iK}[P_K + f_K(t)] + \varepsilon_i(t).$$

It is at this level of the determination of expected returns that the CAPM and the APT really differ. In the CAPM, the expected excess return for an asset is equal to that asset's CAPM beta times the expected excess return on a market index, even for multi-factor versions of the standard CAPM. For such a multi-factor CAPM to be true, the APT risk premia—the P_j 's—must satisfy certain restrictions that are easily derived. In statistical tests these CAPM restrictions have repeatedly been rejected in favor of the APT.

A portfolio manager controls a portfolio's betas—the portfolio's risk exposure profile—by stock selection. Note, e.g., that as the risk exposure to a particular factor is, say, increased, the expected return for that portfolio is also increased (assuming that this risk factor commands a positive risk premium). Thus risk exposures and hence the implied expected returns for a portfolio are determined by a manager's stock selection.

In many applications data are observed monthly, and the 30-day Treasury bill rate is taken as a proxy for risk-free rate, i.e., P_0 in equation (4) is replaced by $TB(t)$, the 30-day Treasury bill rate known to investors at the beginning of month t . Then for a model with N assets ($i = 1, \dots, N$) and a sample period of T time periods ($t = 1, \dots, T$), the data are the asset returns, $r_i(t)$'s, the Treasury bill rates, $TB(t)$'s, and the factor realizations, $f_j(t)$'s. From these data, the statistical estimation problem is to obtain numerical values for the N P_j 's and the $(N \times K)$ β_{ij} 's. Discussion of this econometric problem is beyond the scope of this essay, but we have provided a list of suggested further readings that cover the topic in detail.

IV. The Macroeconomic Forces Impacting Stock Returns

Taking the time period to be one month and using the 30-day Treasury bill rate as a proxy for the risk-free rate of return, our APT model, equation (4), becomes:

$$(5) \quad r_i(t) - TB(t) = \beta_{i1}[P_1 + f_1(t)] + \dots + \beta_{iK}[P_K + f_K(t)] + \epsilon_i(t).$$

From this point there are three alternative approaches to estimating an APT model:

- (i) The risk factors $f_1(t)$, $f_2(t)$, ..., $f_K(t)$ can be computed using statistical techniques such as **factor analysis** or **principal components**.
- (ii) K **different** well-diversified portfolios can substitute for the factors.
- (iii) Economic theory and knowledge of financial markets can be used to specify K risk factors that can be measured from available macroeconomic and financial data.

Each of these approaches has its merits and is appropriate for certain types of analysis. In particular, approach (i) is useful for determining the number of relevant risk factors, that is, for determining the numerical value of K . Many empirical studies have indicated that $K = 5$ is adequate for explaining stock returns. However, the estimates extracted using factor analysis or principal components have an undesirable property which renders them difficult to interpret; this arises because, by the nature of the technique, the estimated factors are non-unique linear combinations of more fundamental underlying economic forces. Even when these linear combinations can be given an economic interpretation, they change over time so that, for example, "factor 3" for one sample period is not necessarily the same combination—in fact it is almost certainly different—than the combination that was "factor 3" in a different sample period.

Approach (ii) can lead to insights, especially if the portfolios represent different strategies that are feasible for an investor to pursue at low cost. For example, if K were equal to two, one might use small- and large-capitalization portfolios to substitute for the factors.

The advantage of approach (iii) is that it provides an intuitively appealing set of factors that admit economic interpretation of the risk exposures (the β_{ij} 's) and the risk premia (the P_j 's). From a purely statistical view this approach also has the advantage of using economic information in addition to stock returns, whereas approaches (i) and (ii) use "stock returns to explain stock returns." This additional information (information about inflation, for example) will in general lead to statistical estimates with better properties, but, of course, insofar as the economic variables are measured with errors, these advantages are diminished.

Selecting an appropriate set of macroeconomic factors involves almost as much art as it does science, though by now it is a highly developed art. The practitioner requires factors that are easy to interpret, are robust over time, and explain as much as possible of the variation in stock returns. Extensive research work—the interested reader is referred to the suggested readings—has established that one set of five factors meeting these criteria is the following:

> $f_1(t)$: Confidence Risk

Confidence Risk is the unanticipated changes in investors' willingness to undertake relatively risky investments. It is measured as the difference between the rate of return on relatively risky corporate bonds and the rate of return on government bonds, both with twenty-year maturities, adjusted so that the mean of the difference is zero over a long historical sample period. In any month when the return on corporate bonds exceeds the return on government bonds by more than the long-run average, this measure of Confidence Risk is positive ($f_1 > 0$). The intuition is that a positive return difference reflects increased investor confidence because **the required yield on risky corporate bonds has fallen relative to safe government bonds**. Stocks that are positively exposed to this risk ($\beta_{i1} > 0$) then will rise in price. (Most equities **do** have a positive exposure to Confidence Risk, and small stocks generally have greater exposure than large stocks.)

> $f_2(t)$: Time Horizon Risk

Time Horizon Risk is the unanticipated changes in investors' desired time to payouts. It is measured as the difference between the return on twenty-year government bonds and 30-day Treasury bills, again adjusted to be mean zero over a long historical sample period. A positive realization of Time Horizon Risk ($f_2 > 0$) means that the price of long-term bonds has risen relative to the 30-day Treasury bill price. This is a signal that investors require a lower compensation for holding investments with relatively longer times to payouts. The price of stocks that are positively exposed to Time Horizon Risk

($\beta_{i2} > 0$) will rise to appropriately decrease their yields. (Growth stocks benefit more than income stocks when this occurs.)

> $f_3(t)$: Inflation Risk

Inflation Risk is a combination of the unexpected components of short- and long-run inflation rates. Expected future inflation rates are computed at the beginning of each period from available information: historical inflation rates, interest rates, and other economic variables that influence inflation. For any month, Inflation Risk is the unexpected surprise that is computed at the end of the month, i.e., it is the difference between the actual inflation for that month and what had been expected at the beginning of the month. Since most stocks have negative exposures to Inflation Risk ($\beta_{i3} < 0$), a positive inflation surprise ($f_3 > 0$) causes a negative contribution to return, whereas a negative inflation surprise ($f_3 < 0$, a deflation shock) contributes positively toward return.

Industries whose products tend to be “luxuries” are most sensitive to Inflation Risk. Consumer demand for “luxuries” plummets when real income is eroded through inflation, thus depressing profits for industries such as retailers, services, eating places, hotels and motels, and toys. In contrast, industries least sensitive to Inflation Risk tend to sell “necessities,” the demands for which are relatively insensitive to declines in real income. Examples include foods, cosmetics, tire and rubber goods, and shoes. Also, companies that have large asset holdings such as real estate or oil reserves may benefit from increased inflation.

> $f_4(t)$: Business Cycle Risk

Business Cycle Risk represents unanticipated changes in the level of real business activity. The expected values of a business activity index are computed both at the beginning and end of the month, using only information available at those times. Then, Business Cycle Risk is calculated as the difference between the end-of-month value and the beginning-of-month value. A positive realization of Business Cycle Risk ($f_4 > 0$) indicates that the expected growth rate of the economy, measured in constant dollars, has increased. Under such circumstances firms that are more positively exposed to business cycle risk—for example, firms such as retail stores that do well when business activity increases as the economy recovers from a recession—will outperform those such as utility companies that do not respond much to increased levels in business activity.

> $f_5(t)$: Market Timing Risk

Market Timing Risk is computed as that part of the S&P 500 total return that is not explained by the first four macroeconomic risks and an intercept term. Many people find it useful to think of the APT as a generalization of the CAPM, and by including this Market Timing factor, the CAPM becomes a special case: if the risk exposures to all of the first four macroeconomic factors were exactly zero (if $\beta_{i1} = \dots = \beta_{i4} = 0$), then Market Timing Risk would be proportional to the S&P 500 total return. Under these extremely unlikely conditions, a stock’s exposure to Market Timing Risk would be equal to its CAPM beta. Almost all stocks have a positive exposure to Market Timing Risk

($\beta_{i5} > 0$), and hence positive Market Timing surprises ($f_5 > 0$) increase returns, and vice versa.⁵

A natural question, then, is: “Do Confidence Risk, Time Horizon Risk, Inflation Risk, and Business Cycle Risk help to explain stock returns better than I could do with just the S&P 500?” This question has been answered using rigorous statistical tests, and the answer is very clearly that they do.⁶

V. Risk Exposure Profiles and the Risks-Return Tradeoff

We have developed a commercially available PC-based software package called the *BIRR*[®] *Risks and Returns Analyzer*[®] (“BIRR” is an acronym for Burmeister, (Roger) Ibbotson, Roll, and Ross) for doing APT-based risk analysis with a model of the sort described here. While econometric estimation of APT parameters (the risk exposures, β_{ij} ’s, and the risk premia or prices, P_j ’s) is beyond the scope of this essay, the suggested readings contain complete discussions of the more technical statistical issues that are involved in parameter estimation. Here we will use the parameters estimated by our *Risks and Returns Analyzer*[®]; the model is re-estimated every month, and our examples here and in the next sections use numbers taken from the April 1992 release, which is based on monthly data through the end of March 1992.

The risk exposure profile for the S&P 500 and the corresponding prices of risk (the risk premia) are:

Risk Factor	Exposure for S&P 500	Price of Risk (%/yr)
Confidence Risk	0.27	2.59
Time Horizon Risk	0.56	-0.66
Inflation Risk	-0.37	-4.32
Business Cycle Risk	1.71	1.49
Market Timing Risk	1.00	3.61

For each risk factor, the contribution to expected return is the product of the risk exposure and the corresponding price of risk, and the sum of the products is equal to the expected return in excess of the 30-day Treasury bill rate:

Risk Factor	Exposure for S&P 500	Price of Risk (%/yr)	Contribution of Risk Factor to Expected Return (%/yr)
Confidence Risk	0.27 ×	2.59 =	0.70
Time Horizon	0.56 ×	-0.66 =	-0.37

Risk			
Inflation Risk	-0.37 ×	-4.32 =	1.60
Business Cycle Risk	1.71 ×	1.49 =	2.55
Market Timing Risk	1.00 ×	3.61 =	3.61
Sum =	Expected Excess Return for the S&P 500 =		8.09

Thus, if the 30-day Treasury bill rate were, for example, 5.00%, your forecasted return for the S&P 500 would be $5.00 + 8.09 = 13.09$ %/yr.

In general, then, for any asset i , the APT risk-return tradeoff defined by equation (3) is:

$$E(r_i) - TB = \beta_{i1}(2.59) + \beta_{i2}(-0.66) + \beta_{i3}(-4.32) + \beta_{i4}(1.49) + \beta_{i5}(3.61)$$

where TB is the 30-day Treasury bill rate. The following four observations will help clarify this risk-return tradeoff.

(i) The price of each risk factor tells you how much expected return will change due to an increase or decrease in your portfolio's exposure to that type of risk. Suppose, for example, that you construct a well-diversified portfolio (call your portfolio p) having a risk exposure profile identical to the S&P 500, except that it has an exposure to Confidence Risk of $\beta_{p1} = 1.27$ instead of $\beta_{S\&P,1} = 0.27$. Since the price of Confidence Risk is $P_1 = 2.59$ %/yr, you will be rewarded for undertaking this additional risk by 1.00×2.59 , i.e., your portfolio will have an expected return that is 2.59 %/yr higher than the expected return for the S&P 500.

(ii) APT risk prices can be negative, and they are for both Time Horizon Risk and Inflation Risk ($P_2 < 0$ and $P_3 < 0$). Consider first Inflation Risk. Because almost all stocks have negative exposures to Inflation Risk (i.e., because their returns decrease with unanticipated increases in inflation), the Inflation Risk contribution to expected return is usually positive—the negative risk exposure times the negative price for Inflation Risk equals a **positive** contribution to expected return. That is, for most i , $\beta_{i3} < 0$, and because $P_3 < 0$, $\beta_{i3} \times P_3 > 0$ for most i .

(iii) However, many stocks have a **positive** exposure to Time Horizon Risk (i.e., $\beta_{i2} > 0$, and thus when the price of long-term government bonds rises relative to the price of 30-day Treasury bills, their return increases). Since the reward for Time Horizon Risk is negative ($P_2 < 0$), this means that for such stocks the Time Horizon Risk contribution to expected return is **negative**, while for stocks with a negative exposure to Time Horizon Risk, the contribution is positive.

Why should this be the case? The answer is that, just as you will pay for an insurance policy that pays off when your house burns down, investors desire to hold stocks whose returns increase when the relative price of long-term government bonds rises. The fact that investors want to hold stocks with this property means that their prices have been driven higher than they otherwise would have been, and, therefore, their expected returns are **lower**. Thus the negative price for Time Horizon Risk produces exactly the result that we want: *ceteris paribus* stocks having larger (positive) exposures to Time Horizon Risk also have lower expected returns.

(iv) Above we report raw values that have not been standardized. We believe that a good approach for judging whether or not a particular value is “significantly different” from another is to plot the actual empirical distribution function across stocks and to visually make an assessment.⁷ Figures 1 and 2 illustrate this empirical distribution function, computed from over 3,200 stocks in the BIRR database, for Business Cycle Risk and the P/E ratio, respectively.

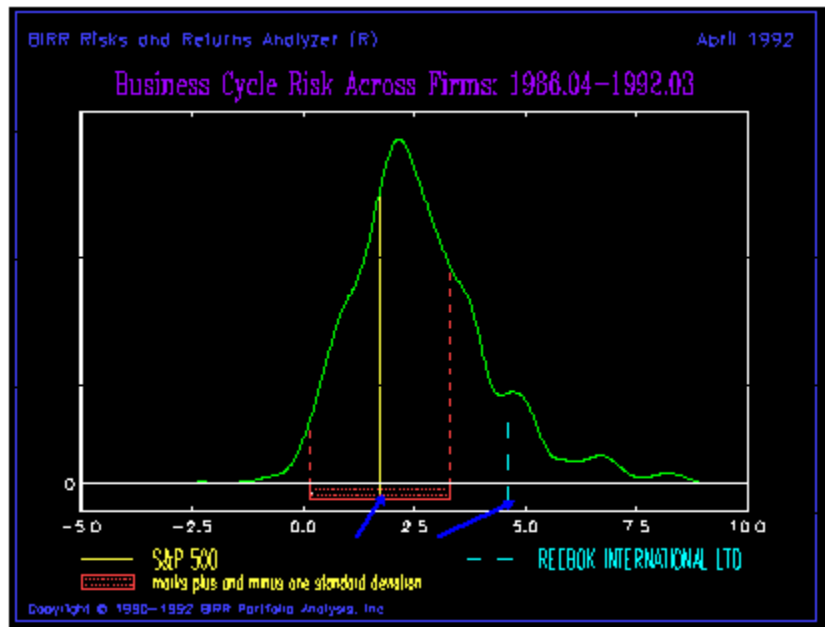


Figure 1. The empirical distribution for Business Cycle Risk (β_{i4}). The β_{i4} for $i =$ the S&P 500 is indicated by the solid vertical line, and the β_{i4} for $i =$ Reebok International Ltd. is indicated by the vertical line with large dashes. The box on the horizontal axis centered on the solid line (the S&P 500) indicates plus and minus one standard deviation of Business Cycle Risk **for all the stocks in the database, i.e., the width of the box is two standard deviations for the across-firm distribution.** Note that the distribution does not appear normal.

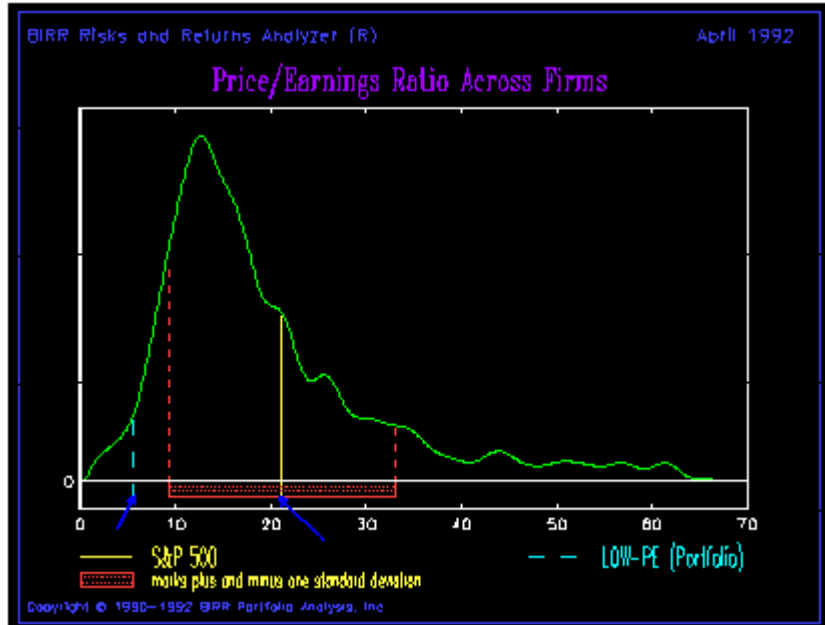


Figure 2. The empirical distribution for P/E ratios. The P/E ratio for the S&P 500 is indicated by the solid line, and the P/E ratio for a market-value weighted portfolio of the fifty lowest P/E stocks listed on the NYSE is indicated by the dashed line. Again, note that the distribution is not normal and appears to be skewed to the right.

As is evident from Figure 1, the Business Cycle Risk for Reebok International Ltd. is much larger than for the S&P 500. The risk exposure profile for Reebok International Ltd. is:

Risk Factor	Exposure for Reebok	Exposure for S&P 500(%/yr)
Confidence Risk	0.73	0.27
Time Horizon Risk	0.77	0.56
Inflation Risk	-0.48	-0.37
Business Cycle Risk	4.59	1.71
Market Timing Risk	1.50	1.00

These exposures give rise to an expected excess rate of return for Reebok equal to 15.71 %/yr, compared with the 8.09 %/yr that we computed for the S&P 500. Figure 3 compares the risk exposure profiles for Reebok and the S&P 500.

In general, the risk exposure profiles of individual stocks and of portfolios can differ significantly. For example, Figures 4, 5, 6, and 7 compare the risk exposure profiles for portfolios of low-capitalization versus high-capitalization stocks, growth stocks versus the S&P 500, a “value” portfolio versus the BIRR stock database, and a growth versus high-yield portfolio, respectively. (The BIRR Risk Index also plotted in these graphs is a single number that gives an **approximate** answer to the question, “Does A have more systematic risk, relative to the market, than B?” The simple derivation of this Risk Index is left as an exercise for the reader.) These risk exposure profiles define **APT styles**, and they enable us to view traditional portfolio management styles (such as small-cap growth managers or large-cap value managers) from a new perspective that reveals their inherent macroeconomic risks.

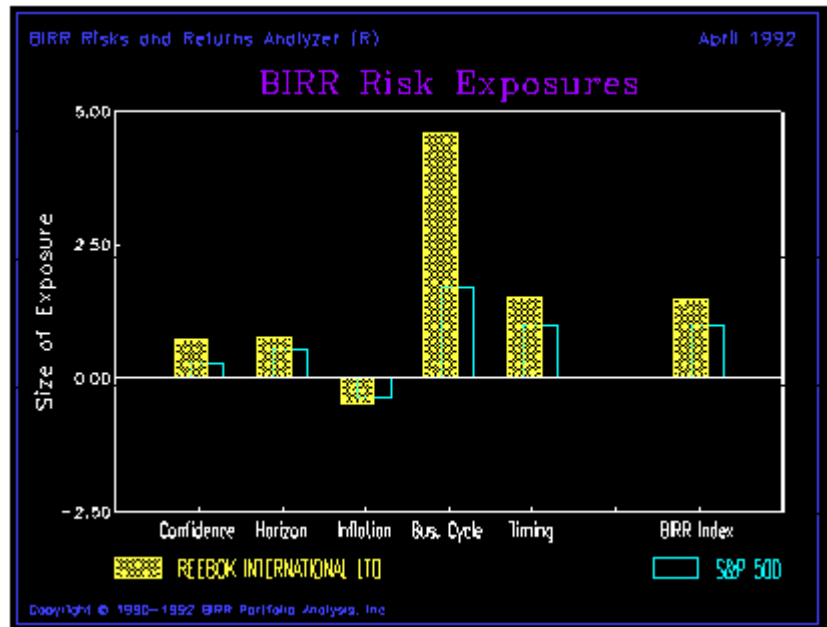


Figure 3. The risk exposure profiles for Reebok International Ltd. and the S&P 500.

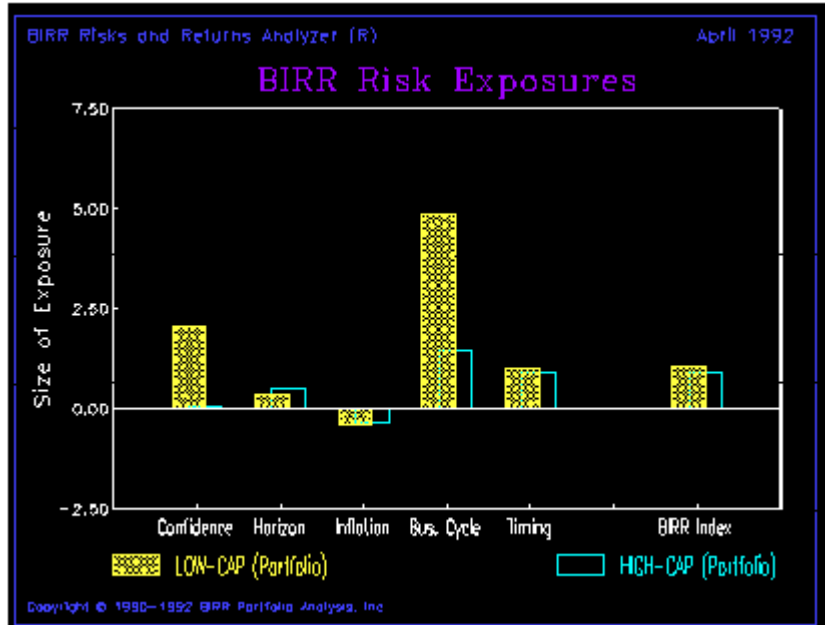


Figure 4. The risk exposure profiles for market-value weighted portfolios of the fifty lowest and highest market capitalization stocks listed on the NYSE.

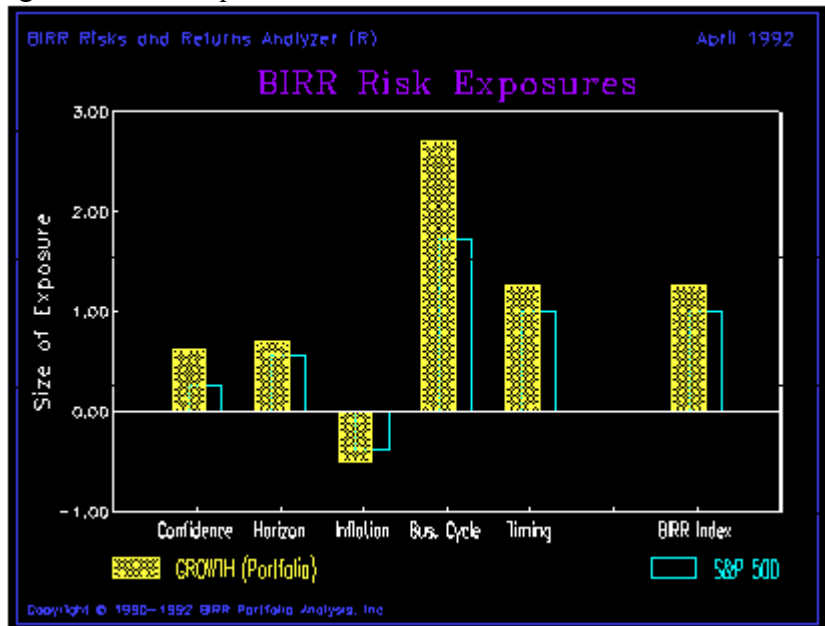


Figure 5. The risk exposure profile for a market-value weighted portfolio of the fifty highest growth rate stocks listed on the NYSE versus the S&P 500.

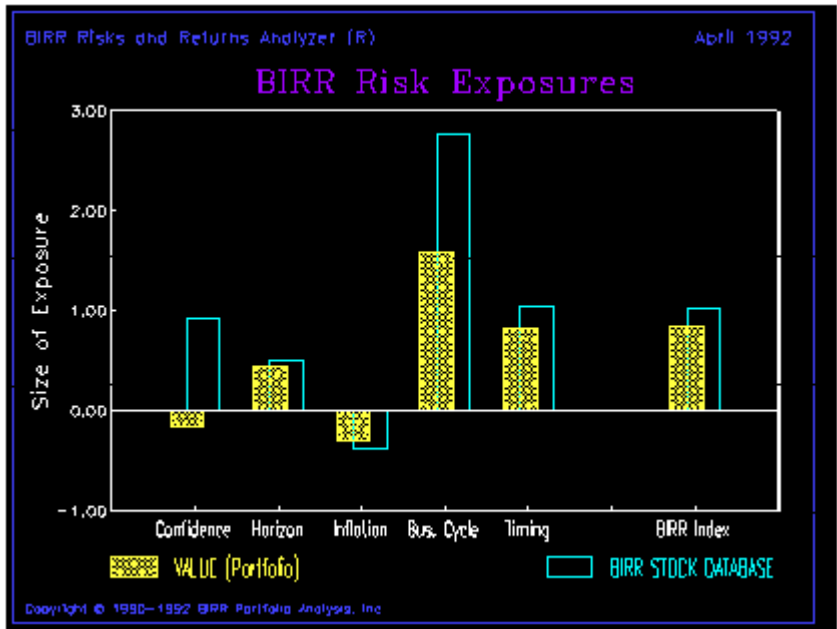


Figure 6. The risk exposure profiles for a market-value weighted portfolio of the fifty lowest P/E stocks, selected from the 500 firms listed on the NYSE with the largest market values (a “value” portfolio), versus an equally-weighted portfolio of all the stocks in the BIRR database.

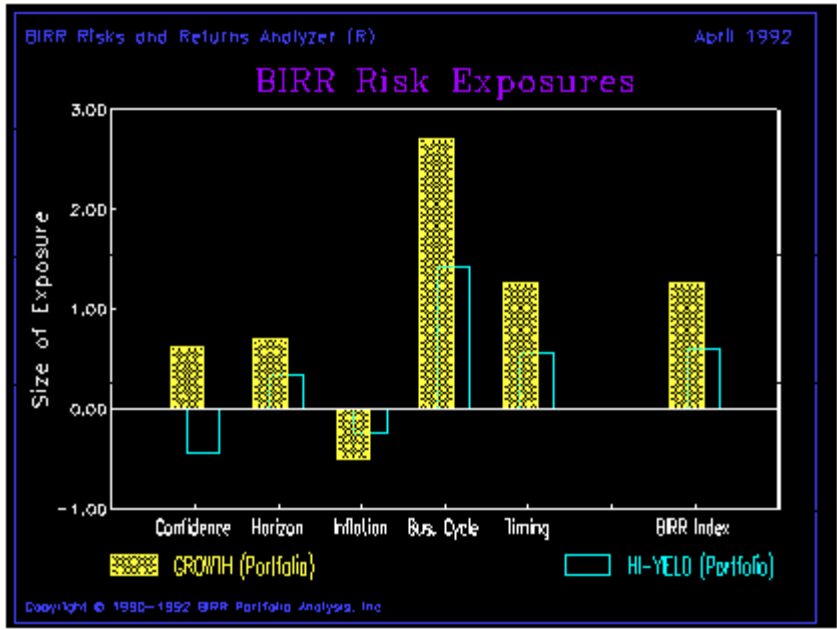


Figure 7. The risk exposure profiles for the growth portfolio in Figure 5 versus a market-weighted portfolio of the 50 highest dividend yield stocks listed on the NYSE.

The usefulness to practitioners of risk exposure profiles and the risk-return tradeoff is an empirical issue. There is abundant evidence showing that market indexes are not mean-variance efficient, and it follows that the usual implementations of the CAPM using some market index as a proxy are invalid. More importantly, recent empirical evidence demonstrates that CAPM betas do not accurately explain returns.

The multifactor APT approach has far greater explanatory power than the CAPM. Numerous econometric studies have verified the superior performance of models which include multiple factors (Postulate 1 of the APT) to explain returns, and which use multiple factor premia (Postulate 2 of the APT) to explain expected returns. These results are discussed in the papers suggested for further reading.⁸

VI. Contributions to Return from Macroeconomic Surprises

The last section talks about the contributions of the macroeconomic factors to expected return. However, surprises always occur, and expected returns differ from actual returns. Taking expectations of equation (5), it follows that the **expected return** for the i -th asset in period t is

$$(6) \quad E[r_i(t)] = TB(t) + \beta_{i1}P_1 + \dots + \beta_{iK}P_K .$$

The expected return given by equation (6) is hardly ever equal to the actual return. Because factors rarely do exactly what is forecast for them and because the idiosyncratic portion of return, $\varepsilon_i(t)$, is almost never zero, the actual return for the i -th asset is

$$(7) \quad r_i(t) = E[r_i(t)] + U[r_i(t)] ,$$

where $U[r_i(t)]$ is the **unexpected return** given by

$$(8) \quad U[r_i(t)] = \beta_{i1}f_1(t) + \dots + \beta_{iK}f_K(t) + \varepsilon_i(t) .$$

Suppose now we consider a historical sample period $t = 1, \dots, T$ and let bars denote sample period means. The **mean *ex post* actual return** for the i -th asset is

$$(9) \quad \begin{aligned} \bar{r}_i &= \bar{E}[r_i] + \bar{U}[r_i] \\ &= \bar{E}[r_i] + \bar{\beta}_{i1}f_1 + \dots + \bar{\beta}_{iK}f_K + \bar{\varepsilon}_i . \end{aligned}$$

That is, the historical mean return for the i -th asset is equal to the sum of the **mean *ex post* expected return** and the mean of the components of return that are surprises.

The **mean *ex post* unexpected macroeconomic factor return** is

$$\bar{\beta}_{i1}f_1 + \dots + \bar{\beta}_{iK}f_K ,$$

and the *ex post* sample period alpha for the i -th asset is

$$\alpha_i \equiv \bar{\varepsilon}_i .$$

Putting all this together, we have that, for the i -th asset, the **mean *ex post* actual return = the mean *ex post* expected return + the mean *ex post* unexpected macroeconomic factor return + α_i** . The first term on the right side measures the rewards for risks; it is the reward received by a manager that is attributable to the risk exposure profile for the portfolio. The second term has two possible interpretations: (i) If a manager has taken intentional macroeconomic bets, the unexpected macroeconomic factor return measures the success or failure of those bets (e.g., a “bet” on an economic expansion through an unusually large exposure to Business Cycle Risk), but (ii) if a

manager is not intentionally making factor bets, it can be interpreted simply as a measure of good or bad luck in this sample period. Finally, the last term, α_i , is a measure of a manager's selection of individual stocks which perform better or worse than *a priori* expectations and is our measure of **APT selection**.

Since by construction all of the macroeconomic factors have zero population means (that is, they have zero-mean probability distributions), over long historical periods their sample means will be approximately zero.⁹ Thus over long historical sample periods, the contribution to return from the macroeconomic surprises will be approximately zero. Over long time periods, then, almost all of the mean realized return will be due to the rewards for risks, and, possibly, APT selection.

However, over short time periods this will not be the case even for managers with no timing skills; the surprises arising from the macroeconomic factors can have significant impacts on realized returns, as illustrated below for Reebok International Ltd. and the S&P 500.

	Mean <i>Ex Post</i> Unexpected Macroeconomic Factor Return:	
Sample Period	Reebok	S&P 500
Apr 1991 to Mar 1992 (12 months)	-2.03 %/yr	-1.58 %/yr
Oct 1990 to Mar 1992 (18 months)	14.24	9.31
Apr 1990 to Mar 1992 (24 months)	-0.95	-0.86
Apr 1987 to Mar 1992 (60 months)	-4.01	-2.95
Apr 1986 to Mar 1992 (72 months)	-0.26	-0.56

VII. How to Use the APT: Some Examples

A primary concern for practitioners is not only to acquire an understanding of the APT, but also to learn how to use it to enhance their investment performance. So far we have concentrated on explaining the APT, and now we will briefly discuss several uses of the APT that every practitioner could easily employ. The following list is chosen to be exemplary of some more widely used APT techniques, but it is by no means exhaustive.

> Evaluation of Macroeconomic Risk Exposures and Attribution of Return

We have seen that risk exposure profiles can vary widely for stocks and portfolios. These risk exposure profiles are determined by the risks a manager undertakes through stock selection, and they determine a manager's **APT style**.

A basic first task, then, is to determine the risk exposure profiles for your portfolios. Usually a manager will want to compare his or her risk exposure profiles with those for an appropriate benchmark. Suppose, for example, that you are a small-cap manager. You should know whether or not your portfolio differs in its exposure to macroeconomic risks from an appropriate index of small-capitalization firms. If you find differences, that fact alone will account for performance differentials from the index. Only if you find that your risk exposures are the same as the index can you attribute *ex post* superior performance to your **APT selection**, i.e. to your selection of individual stocks that returned more than would be expected on the basis of the risks undertaken.

Whatever your risk exposure profile, you should use the APT to divide your mean *ex post* actual return into: (i) expected return, which is your reward for the risks you took, (ii) unexpected macroeconomic factor return, which arises from your factor bets and factor surprises, and (iii) your α , which arises from stock selection. Moreover, as we have seen, expected and unexpected factor return can be attributed to your risk exposure profile. Thus this analysis will give you a better understanding of the true sources of your actual portfolio performance.

> Index Portfolios

A closely related topic is the formation of index portfolios designed to track particular well-diversified benchmarks. The APT provides powerful tools for tracking **any** such benchmark portfolio. A tracking portfolio can be constructed simply by forming a portfolio with a matching risk exposure profile. Your *ex post* APT α can be made small by making your tracking portfolio well-diversified so that your portfolio-specific return, call it ϵ_p , is near zero.

It is more difficult to track a benchmark that itself is not well-diversified in the sense that its *ex post* α usually is not near zero. In this case you must not only match risk exposure profiles, but you also must match the benchmark's α . One way you might do this is to form your tracking portfolio by random sampling from the stocks that constitute the benchmark.

> **Tilting: Making a Factor Bet**

Good managers may possess superior knowledge about the economy. Suppose, for example, you believe that the economy is going to recover from a recession faster than most market participants. If you are correct in this belief, the realizations of Business Cycle Risk will be positive [$f_4 > 0$], and stocks which have greater risk exposures to Business Cycle Risk [stocks for which β_{i4} is larger] will, *ceteris paribus*, outperform.

To take advantage of your superior knowledge, you will want to make a factor bet on [or tilt toward] Business Cycle Risk. That is, you will want to alter your existing portfolio to increase its Business Cycle Risk exposure without changing any other macroeconomic risks.

Conversely, if you have special knowledge that the economy is going to slide into a recession, you will want to lower your exposure to Business Cycle Risk.

> **Multi-Manager Fund Performance**

Most sponsors employ more than one manager. Even though each individual manager may perform well when compared with his or her particular style benchmark, that is not the issue of most importance to the sponsor. You as a sponsor want to evaluate the risks and performance of your overall fund.

You should combine the portfolios of your individual managers into one overall fund portfolio. You then can use the APT to examine the risk exposure profile and performance of this fund portfolio. Often you may find that the combination of managers leads you to risk exposures with which you are uncomfortable. It then is usually a simple task to find a reallocation of funds among your managers that achieves the fund risk exposure profile you desire.

Likewise, you can examine whether or not your overall fund return exceeds your benchmark portfolio and determine the sources of differences.

> **Optimized Risk Control with Manager-Supplied Rankings**

Many managers have their own proprietary methods for evaluating stock return performance, yet lack adequate methods for estimating their accompanying risks. The APT, or more accurately part of the APT, is a perfect tool for such managers.

To keep matters simple, suppose that you have your own ranking system that scores every stock on a scale from one to ten, where ten is the score given to the stocks in the best expected return category. Your objective is to emulate the volatility of the S&P 500, but to achieve a higher return. How could you use the APT?

Let s_i be the score from one to ten assigned by you to the i -th stock, $i = 1, \dots, N$. Your formal problem is to find portfolio weights w_1, w_2, \dots, w_N for the N stocks in your selection universe such that you maximize your portfolio score, yet have a risk exposure profile that is similar to the S&P 500. More formally, you want to find weights that give you the highest possible value for

$$w_1 \times s_1 + w_2 \times s_2 + \dots + w_N \times s_N$$

subject to the constraints that your portfolio betas,

$$\beta_{pj} = w_1 \times \beta_{1j} + w_2 \times \beta_{2j} + \dots + w_N \times \beta_{Nj}$$

for $j = 1, \dots, K$, are close to the betas for the S&P 500. That is, you want to find weights that make the risk exposure profile for your portfolio close to the risk exposure profile for the S&P 500, while at the same time maximizing the value of your portfolio's ranking score. If your ranking system works, you then will achieve a return that is superior to the S&P 500. If the resulting portfolio is well-diversified, it and the S&P 500 will have approximately equal volatilities. The proper diversification can be achieved by making N sufficiently large and by imposing a maximum value for your w 's so that your portfolio contains a large number of stocks.

This optimization problem is easily solved using linear programming.

> Long-Short Investment Strategies

Long-short or market neutral investment strategies have received increased attention. We first will discuss the pure APT view of such strategies, and then show how the APT can be used to effectively implement them by managers with superior knowledge.

Suppose you hold a long portfolio with return $r_L(t)$ and a short portfolio with return $r_S(t)$, and that both have equal dollar values. Let the risk exposures for these portfolios be denoted by β_{Lj} and β_{Sj} , $j = 1, \dots, K$. Assuming that you earn the 30-day Treasury bill rate on the proceeds from the short position, your total return is

$$r_L(t) - r_S(t) + TB(t).$$

Let the risk exposure profile on your long portfolio exactly match the risk exposure profile on your short position. Then, using equation (3), the **expected** returns on your long and short portfolios are equal, and thus the expected return for your long-short strategy is simply $TB(t)$, while the variance of your realized return is

$$\text{var}[\epsilon_L(t) - \epsilon_S(t) + TB(t)].$$

Since no stock is held in both the long and short portfolios, this variance is approximately

$$\text{var}[\epsilon_L(t)] + \text{var}[\epsilon_S(t)] + \text{var}[TB(t)].$$

The position has greater volatility than 30-day Treasury bills, but no greater mean return, and therefore it is not a very attractive strategy, particularly after trading costs.

It could become attractive, however, if the APT alphas on the long position were significantly larger than the APT alphas on the short position, i.e., it is an attractive strategy for a manager with superior **APT selection**. Imagine that you are one of these exceptional managers who can pick two well-diversified portfolios of stocks, with no stocks in common, such that $\alpha_L > 0$ for the long portfolio and $\alpha_S < 0$ for the short portfolio. **If** you also can match the risk exposure profiles of your long and short positions, you can earn a return $\alpha_L - \alpha_S + TB(t)$ with a volatility approximately equal to that of 30-day Treasury bills.

The APT can play a crucial role for such a manager: it provides an easy and quick way to match the risk exposure profiles of the long and short positions. To exemplify this role of the APT, we constructed a long portfolio consisting of approximately fifty stocks on the NYSE with the largest *ex post* alphas over a sample period of 72 months (April 1986 to March 1992). We then computed the risk exposure profile for this long portfolio. A short portfolio of approximately fifty stocks was selected by choosing from the NYSE, but excluding the contents of the long portfolio. An optimization problem was solved to find portfolio weights for the short position that matched its risk exposure profile to the long position. The resulting risk exposure profile for the overall long-short strategy is illustrated in Figure 8; it has essentially zero systematic risk. The sole source of volatility (beyond the volatility of 30-day Treasury bills) for this long-short strategy comes from the ϵ 's for the long and short positions; by having portfolios of fifty stocks or more, this volatility can be kept small.

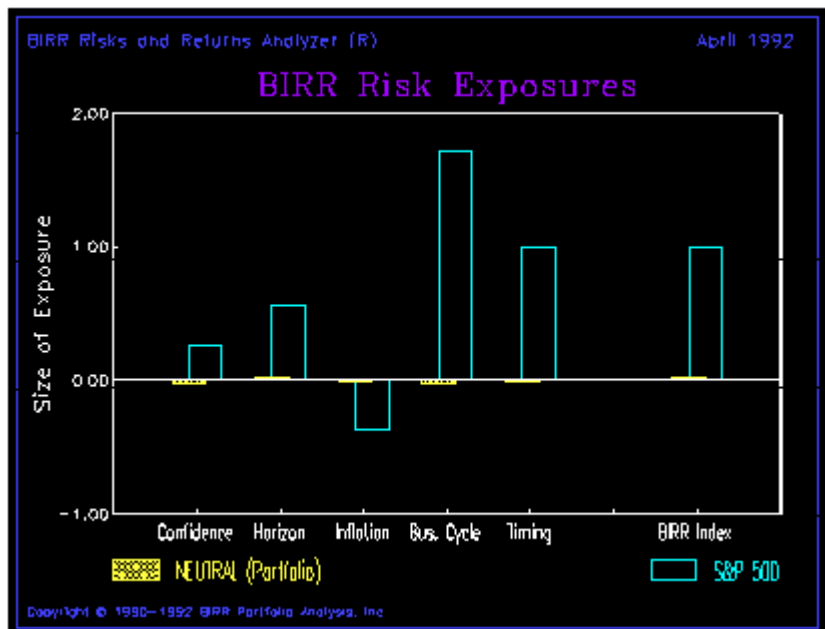


Figure 8. The risk exposure profile for the market neutral strategy described in the text. Its risk exposure profile is Confidence Risk = -0.02, Time Horizon Risk = 0.02, Inflation Risk = 0.00, Business Cycle Risk = -0.02, and Market Timing Risk = 0.00 [with a BIRR Risk Index of 0.01].

The performance of this long-short or market neutral strategy over the most recent twelve months of the sample period (April 1991 to March 1992) is illustrated in Figure 9. The mean realized return was 30.04 %/yr, compared with 11.57 %/yr for the S&P 500. However, the standard deviation of this realized return was only 6.26 %/yr, compared with 18.08 %/yr for the S&P 500.

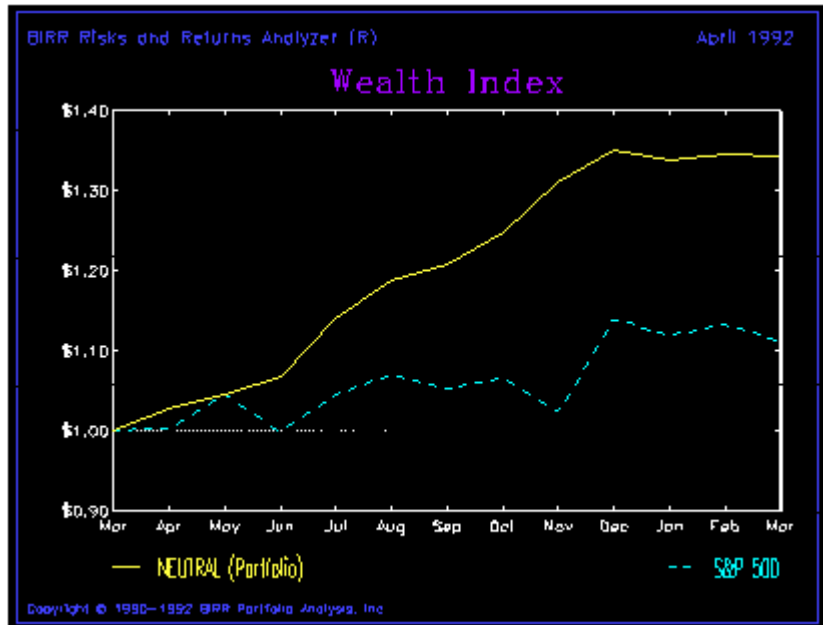


Figure 9. The cumulative wealth for the market neutral strategy discussed in the text compared with the S&P 500. The graphs show the cumulative wealth from \$1.00 invested at the beginning of March 1991.

> Mean-Variance Efficiency

The standard optimization problem of finding the portfolio with the highest expected rate of return for a given variance is easily solved within an APT framework. For this problem the expected return could either be given by the APT's equation (3), or it could come from manager-supplied rankings as discussed above. In either case, there are a variety of computational methods that can be used to calculate the optimal portfolio weights. In such problems one often takes the APT's systematic variance [rather than total variance] as given and then imposes constraints to assure that the resulting portfolio is well-diversified. This procedure often produces superior results because estimates of stock return variances and, especially, covariances tend to have large out-of-sample errors.

VIII. Concluding Remarks

What we have described in this essay is the foundation for the more sophisticated portfolio management techniques that are opened up by the APT. It is our hope that a careful reading will enable the practitioner to apply the APT to the construction of superior portfolios and will help to provide an understanding of the true sources of actual return. In contrast to other common return measurement approaches, the APT offers fewer explanations for return differences. This simplicity is a great virtue. We think it is far easier to understand the true sources of stock returns with an APT model than with models having dozens and dozens of parameters that supposedly influence returns.

The basic APT model described here can be enhanced in many ways. Some of the generalizations now in use include the following:

- Allowing the risk prices, the P_j 's, to vary over time.
- Allowing the risk exposures, the β_{ij} 's, to vary over time.
- Use of Bayesian methods to produce optimal out-of-sample forecasts for the risk exposures and hence for the expected returns.
- Introduction of additional factors with zero risk prices which are typically used to capture industry and sector effects. While such non-priced factors do not contribute to expected return, they do help to explain volatility, and they provide managers with a tool to evaluate the diversification of their portfolios.

Other enhancements are being invented every day, and as more and more tools become available and as understanding of the APT spreads, so does the application of the APT to portfolio management problems.

IX. References and Further Reading

For more information on the APT, you may wish to read some of the items on the following reading list. This list is by no means exhaustive and is meant only as a guide to get started. The list is composed of three categories, based on the level of technical detail contained in the reading.

A. Introductory Expositions

Berry, Michael A., Burmeister, Edwin, and McElroy, Marjorie B., "Sorting Out Risks Using Known APT Factors," *Financial Analysts Journal*, 29-42, March/April (1988).

Berry, Michael A., Burmeister, Edwin, and McElroy, Marjorie B., "A Practical Perspective on Evaluating Mutual Fund Risk," *Investment Management Review*, 2(2), 78-86, March/April (1988).

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X. Footnotes

¹ Revised March 1997, August 2001, and March 2003. This paper is based on an early version of “A Practitioner’s Guide to Arbitrage Pricing Theory,” a contribution to *A Practitioner’s Guide to Factor Models* for the Research Foundation of the Institute of Chartered Financial Analysts, 1994.

² More precisely, if $r_m(t)$ is the return (in time period t) on a market index such as the S&P 500, the CAPM measure of the riskiness for asset i with return $r_i(t)$ is equal to that asset’s CAPM beta defined by $\beta_i = \text{cov}[r_i(t), r_m(t)]/\text{var}[r_m(t)]$.

The CAPM is equivalent to the statement that the market index is itself mean-variance efficient in the sense of providing maximum average return for a given level of volatility. The index used to implement the CAPM is implicitly assumed to be an effective proxy for the entire market of assets.

³ Of course, the “all other things equal” can only be expected to hold on average over many time periods.

⁴ An equivalent interpretation of equation (3) uses an analogy to the familiar relationship that “quantity \times price = value.” Thus if we think of β_{ij} as **the quantity of type-j risk in the i -th asset** and P_j as **the price of type-j risk**, then the product $\beta_{ij}P_j$ is **the value of the contribution of type-j risk to the expected return of the i -th asset**. If we let V_{ij} denote this value, then it follows from equation (3) that the sum of all the values is equal to the expected excess return (the expected return in excess of the risk-free rate) for the i -th asset, i.e.,

$$E[r_i(t)] - P_0 = V_{i1} + \dots + V_{iK} .$$

⁵ Market Timing Risk is not required in an APT model that includes **all** the relevant macroeconomic factors. However, as a practical matter, some relevant macroeconomic factor may be difficult to measure or may not even be observable. Market Timing Risk will capture the effects of any such unobserved macroeconomic factor.

⁶ The probability that the first four macroeconomic factors do not add any information that is useful for explaining stock returns is less than the probability that a standard normal variable (a random variable that is normally distributed with mean zero and standard deviation one) exceeds 20 in value, i.e., it is virtually zero.

⁷ It is not uncommon for the numerical values of financial attributes (such as P/E ratios) to be reported in units of standard deviation. That is, a standardized value is computed by first transforming the variable so that it has mean zero and unit variance; a standardized value of 1.0 (-1.0) means that it lies one standard deviation above (below) the mean. **Provided** the attribute is distributed normally, 68.26% of the observations lie between the standardized values -1.0 and 1.0, 95.44% lie between -2.0 and 2.0, etc. However, the use

of such standardized values can be misleading and even dangerous if the underlying financial attribute is not distributed normally.

⁸ Econometric testing is a difficult subject. Some of the technical issues include:

- The sample of stocks selected.
- The length of the time period and the frequency of the data.
- The exact specification of the model, including how the factors are measured.
- The technical issue called **identification**, which, loosely speaking, is a way of examining whether or not all of the parameters can be statistically estimated. For example, suppose that you have a model with parameters a , b , and c , related by $ab = c$, and that you have estimated that $c = 2$; without additional information, the parameters a and b are **not identified**.

The technical issue of **multicollinearity** that arises when two or more variables that are being used to explain asset returns are themselves approximately linearly related, in which case the parameter estimates are subject to large errors and are unstable from sample to sample. If some *ad hoc* rule is used to stabilize the estimates across samples, out-of-sample forecast errors will be unacceptably large.

- The exact statistical technique used to compute the parameter estimates.
- The specific alternative against which the model is to be compared.

⁹ This statement is literally true for a portfolio with constant betas. It is possible, however, that “timing” managers can successfully alter betas from period to period so that the average contribution of the factor surprises to portfolio returns is **not** zero. For instance, if managers can predict changes in real business activity (as measured, say, by industrial production) better than the market as a whole, they could structure their portfolios to have high (low) business cycle exposure when they predict an increase (decrease) in business activity.